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PERVC METHOD FOR DETERMINATION OF VARIANCE COMPONENTS

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Abstract. Among the problems appearing in the mathematical treatment of the observational material is the one of weights which is still unresolved because of the unknown structure of observational variances. In this paper a method referred to as the PERVC one is presented. Its purpose is the estimation of the variance components and it is tested on three models of variance components. These models were used in the adjustment of the part of the European Longitude Network (ELN) undertaken in the framework of a proposal for the project of including Belgrade in this network. The results of the estimation of variance components obtained by using this method are compared to those where the MINQE one is applied.

1. INTRODUCTION

One of the main tasks in the analysis of observations is the examination of the mathematical model of adjustment both linear and stochastic. The examination of the stochastic model, i.e. of the model of weights, is equivalent to the examination of the model for the variance components (VC) of observations. In solving this task we considered three VC models. For the purpose of estimating the variance components in the practise the MINQE method has been used most frequently and the estimates obtained by applying this method for all the three models can be found in the paper by Perović and Cvetković (2001).

In this paper the same three models of variance components of observations are used, but the components are estimated by applying the PERVC method (Perović, 2005). The obtained PERVC estimates are compared with the corresponding MINQE ones. In this study we use the observational material analysed in papers (Perović and Cvetković, 1988; Cvetković and Perović, 1999; Cvetković and Perović, 2000; Perović and Cvetković, 2001).

2. MATHEMATICAL MODEL

In the model adjustment for longitudes we use the Gauss-Markoff model for the variance-covariance analysis:

$$\begin{array}{ll} (a) & \text{Linear}: \mathbf{v} = \mathbf{Ax} + \mathbf{Bt} + \mathbf{f}, \quad \mathbf{f} = \mathbf{l}_o + \mathbf{l} \\ (b) & \text{Stochastic}: \mathbf{M}[\mathbf{v}] = \mathbf{0}\ \mathbf{M}[\mathbf{vv}^T] = \mathbf{K} = \sigma^2\mathbf{P}^{-1} = \sigma^2 diag\left\{P_i^{-1}\right\} \end{array} \tag{1}$$

where $\mathbf{v}$ is the vector of measurement corrections, $\mathbf{l}$ is the vector of measurements, $\mathbf{l}_o$ the one of approximate values for the measured quantities, $\mathbf{x}$ the vector of basic parameters, $\mathbf{t}$ the one of additional parameters; $\mathbf{A}$ and $\mathbf{B}$ are the matrices of known coefficients, σ^2 is the variance coefficient; $\mathbf{K}$ is the variance-covariance matrix of measurements and $\mathbf{P}$ is the one of measurement weights.

In the examination of the VC model we also study the functional models (Perović and Cvetković, 2001). The best functional model is used in the accuracy examination for the PERVC method.

The examination of the weight models for the measured zenith distances z is reduced to the examination of the model of the variance components (VC) for the zenith-distance measurements.

1^o **Wende's model of variance components (WVC)**. Wende accepted the same variance value for all zenith-distance observations concerning the stars from the same group (Wende, 1992). So in the adjustment of the observations he used three different variance values only, one in each group of observed stars, 10, 11 and 12, i.e.

$$\sigma_z^2 = \sigma_i^2, \qquad i = 10, 11, 12\ . \tag{2}$$

2^o **Two-component variance model (VC2)**. The variance in the zenith distance is given by the formula:

$$\sigma_z^2 = \sigma_1^2 + \sigma_2^2 \cdot Q_2, \quad (Q_2 = (\cos\varphi \sin\mathrm{A})^2), \tag{3}$$

on the basis of which the variance components σ_1^2 and σ_2^2 should be estimated. Q_2 is the velocity of motion of the star observed.

3^o **Three-component variance model (VC3)**. Examining the dependence of the variances σ_t^2 of the random errors in the time recording on individual regressors the authors find a linear dependence σ_t on the apparent magnitude of stars m_v. Due to this in equation (4) another variance component is introduced so that for σ_z^2 one obtains a three-component model:

$$\sigma_z^2 = \sigma_1^2 + \sigma_2^2 \cdot Q_2 + \sigma_3^2 \cdot Q_3, \quad (Q_2 = (\cos\varphi \sin\mathrm{A})^2, Q_3 = m_v^2)\ . \tag{4}$$

3. PERVC METHOD

The Gauss-Markoff model can be described as a general mixed linear model:

$$
\begin{aligned}
\mathbf{l} &= \mathbf{l}_o + \mathbf{A}\mathbf{x} + \varepsilon, \quad \varepsilon = \mathbf{F}_1\xi_1 + ... + \mathbf{F}_k\xi_k \ , \\
\varepsilon &= N[\mathbf{0}, \mathbf{K} = \sigma_1^2\mathbf{Q}_1 + ... + \sigma_k^2\mathbf{Q}_k] \ ,
\end{aligned}
\tag{5}
$$

where $\mathbf{Ax}$ corresponds to $\mathbf{Ax} + \mathbf{Bt}$ from (1); $\mathbf{F}_j, \ j = 1, ..., k$, are known matrices; ξ_j are the vectors of unobserved (latent) measuring errors such that $\xi_j \sim N[\mathbf{0}, \sigma_j^2\mathbf{E}], \ j = 1, ..., k, \mathbf{K}(\xi_i, \xi_j) = \mathbf{0}$ for $i \neq j$; a $\sigma_1^2, ..., \sigma_k^2, \ (\sigma_j^2 > 0), k < n$, the variance components to be estimated; $\mathbf{Q}_j = \mathbf{F}_j\mathbf{F}_j^T, \ j = 1, ..., k$, are known cofactor matrices of the hypothetical measuring errors and $\mathbf{K}$ is the variance-covariance matrix for the vector of measuring errors ε.

This model is called the model of variance components, or briefly VC model.

There are several methods used in estimating the variance components. The MINQE (MInimum Norm Quadratic Estimation) method, established by Rao in (1970), is well known and completely based from the scientific point of view. However, this method requires the application of quadratic matrices of order n resulting in a large memory space of a computer. For this reason one searches in the applications methods free from this disadvantage. The PERVCV1 one (Perović, 2005) is among such methods. This is **PER**ović's method for estimating the **V**ariance **C**omponents by means of the correction estimates **v**, where the digit **1** at the end means the *first method*. It is based on the stochastic asymptotic theory which for well structured linear models, with $n \gg r$, yields good results (the estimates of variance components are close to the true values).

The basic assumption in the PERVCV1 method is that the linear model is well structured and, as a consequence, we have

for a sufficiently large n: $\mathbf{v} \approx -\varepsilon$ and $\mathbf{K}_{\hat{\mathbf{v}}} \approx \mathbf{K}$.

So, by replacing the variance-covariance matrix of the estimates for the corrections of measurements $\mathbf{K}_{\hat{\mathbf{v}}}$ by the variance- covariance matrix $\mathbf{K}$ of the measurement errors the VC model becomes:

$$
\begin{aligned}
&\mathbf{M}[\hat{\mathbf{v}}] = \mathbf{0}, \ \ \mathbf{K}_{\hat{\mathbf{v}}} \ \text{ substituted by } \ \sigma_1^2\mathbf{Q}_1 + \ + \sigma_k^2\mathbf{Q}_k \ , \\
&\mathbf{Q}_j = diag\left\{Q_{j,ii}\right\}, \ j = 1, 2, ..., k; \ i = 1, 2, ..., n \ .
\end{aligned}
\tag{6}
$$

In order to estimate the variance components σ_1^2, σ_2^2 , , σ_k^2 we apply the iterative procedure.

4. RESULTS

The variance components for all the three VC models (WVC, VC2 and VC3) determined by applying the PERVCV1 method are presented in Table 1. In these tables we also give the corresponding VC estimates obtained earlier by applying the MINQE method (Perović and Cvetković, 2001).

Table 1: VC estimates according to: a - WVC model, b - VC2 model and c - VC3 model; $f = 1272$ is number of degrees of freedom

	Method	PERVCV1	MINQE	relative difference
a	$\sigma_{10}^2['']$	0.017704	0.015492	14.3%
	$\sigma_{11}^2['']$	0.011060	0.016709	33.8%
	$\sigma_{12}^2['']$	0.020399	0.016069	26.9%
b	$\sigma_1^2['']$	0.007652	0.008975	14.7%
	$\sigma_2^2['']$	0.019301	0.019156	0.8%
c	$\sigma_1^2['']$	0.003564	0.004828	26.2%
	$\sigma_2^2['']$	0.020648	0.019495	5.9%
	$\sigma_3^2['']$	0.0001939	0.0002073	6.3%
VC2	$\sigma_z^2['']$	0.017308	0.018553	6.7%
VC3	$\sigma_z^2['']$	0.016990	0.017892	5.0%

Using the estimates of the variance components for the VC2 and VC3 models we calculate the variances of observations σ_z^2 according to formulae (3) and (4). So, for instance, for $\varphi = 45°$, A = 80° and $m_v = 4.0$ (Table 1) the relative difference of observation variances obtained by applying two methods (MINQE and PERVC) is a few percents. In the case of WVC this difference is slightly larger which is due to one component. For this model the structure of observation variance is not prominent so the comparison of the results obtained by applying two methods, MINQE and PERVC, is meaningful only in the cases of VC2 and VC3.

5. CONCLUSION

For both VC models (VC2 and VC3) the estimates of the variance components obtained by using the PERVC method agree well with those obtained by using the MINQE one. Among the reasons is the large number of measurements n = 1377, i.e. a large number of degrees of freedom f = 1272. The relative difference between the MINQE and PERVC estimates is lower in the case of VC3 than in the case of VC2. This is due to a better description of the structure of the observation variances by means of the VC3 model. Apart from, the application of the PERVC method requires a shorter time and a smaller computer memory and the results agree well with those for MINQE. Consequently, PERVC is more practical in the case where we have a large number of measurements.

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