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SPACING OF PLANETS IN AN EFFECTIVE GRAVITATIONAL ACCRETION MODEL

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Abstract. Planetary system formation is studied using numerical simulations of an effective model of gravitational accretion. The spacing of planets resulting from the model is compared with Solar and extrasolar planetary systems data.

1. INTRODUCTION

The general planetary formation scenario starts with a protostar which contracts and as a result of centrifugal instability forms a disk. The disk carries with it most of the original angular momentum of the protostar. The remaining material at the center is then free to continue contracting and eventually to become a star. The material making up the disk, on the other hand, is acted upon by two different types of processes – depletion and accretion. Depletion consists of: the effects on the disk of the explosive birth of the star, photo-evaporation and the in-fall of material from the disk onto the star. Accretion, i.e. the creation of ever more massive condensates, may be due to gravitational perturbations, or the formation of instabilities, or the crossing of eccentric orbits, etc.

The general predictions of this scenario are in agreement with the fact that: (a) planetary systems are common, (b) planetary orbits are co-planar and prograde, and (c) the chemical composition of planets is such that light and rocky planets are near the star, while massive and gaseous planets are further out. The problem, until a decade ago, was that we had a plethora of detailed models but the only planets we could see were those in the Solar system. Not surprisingly all the detailed models predicted that all planetary systems will be much like our own. In fact these were models with many phenomenological parameters that were tweaked so as to explain the details of our own planetary system. Few effective models were analyzed, practically no investigation was made of the effect of different initial conditions in the disk on the products of accretion. Not surprisingly we found that detailed phenomenological models, while extremely useful in a narrow and precisely defined setting, turn out to have little predictive power when applied to a wider setting. For example, detailed numerical simulation models of Earth weather are practically useful (say) for predicting where hurricanes will go, however, they utterly fail to tell us about the atmospheric conditions on Venus or Mars. In this sense these models are heuristically sterile - they do not link to related phenomena.

Then, in the early 1990s the first extrasolar planets were discovered. At first their number seemed to grow exponentially, opening up the hope that observation will, on its own, solve the riddle of the formation of planetary systems. For example, in 2000 it seemed that by 2010 we will have detected around 10^4 planets. The detection of extrasolar planets is truly a huge feat of astronomical observation, yet if we ponder extrapolations today we see a growth that is much slower than exponential. Rather than having 10^4 planets to play with in 2010 it seems that we will have to make due with around 200 of them, i.e. not many more than we see today. It turns out that the extrasolar planets we know today are, for the most part, weird. They are very massive, with high eccentricity, often very near the star. Despite selection effects of detection methods it is important to stress that previous models did not predict the possibility of such planetary systems. For a detailed review of extrasolar planets see Perryman 2000.

Instead of putting more parameters in the old models, we believe that the next step needs to be the construction of effective models that link together different accreting phenomena such as planet formation, binary system formation, and disk remnants in a unique theoretical framework with a goal of understanding qualitatively what is common to gravitational accretion in general. This approach has an important potential payoff for planet formation since it links it to excellent short term observational prospects such as the study of binary systems and circumstellar disks. In the next section we will present one such effective model and investigate some of its properties.

2. AN EFFECTIVE MODEL OF ACCRETION

We will investigate the spacing of the planets using a simple effective gravitational accretion model (Balaž et al., 1999abc). The model involves the following simplifications: between interactions matter moves along circular Keplerian trajectories inside a planar disk; interactions are instantaneous (once a given interaction criterium is satisfied); the effect of the interaction between a particle at position r_1 with mass m_1 and spin s_1 with another particle at r_2 with mass m_2 and spin s_2 is that they form a new particle at r with mass m and spin s. The new quantities follow from the old ones from three conservation principles. Conservations of mass and energy give

$$m = m_1 + m_2 \ , \qquad\qquad \frac{m_1 + m_2}{r} = \frac{m_1}{r_1} + \frac{m_2}{r_2} \ . \tag{1}$$

Here we have neglected the energy of the gravitational interactions between particles, as well as thermal energies at their collisions. Conservation of angular momentum determines the new spin to be

$$s = s_1 + s_2 + m_1\sqrt{GM}\sqrt{r_1} + m_2\sqrt{GM}\sqrt{r_2} - (m_1 + m_2)\sqrt{GM}\sqrt{r} \ . \tag{2}$$

Dynamics, i.e. the interaction criterion, follows from what is essentially a dimensional analysis of Newton gravitation. Interaction happens if $F\triangle t \gtrsim |\triangle\vec{p}|$, where F is the gravitational force between the two bodies, $|\triangle\vec{p}|$ the change in momentum before and after collision, and $\triangle t \sim |\vec{r_2} - \vec{r_1}|/|\vec{v_2} - \vec{v_1}|$ is the characteristic time of collision. The simplifying assumption is that before and after the collision particles move on circular Keplerian orbits (in principle this may be relaxed to include elliptic orbits). This leads to a simple (algebraic) interaction criterion

$$\frac{1}{m_1 m_2}\left|\frac{m_1+m_2}{\sqrt{r}} - \frac{m_1}{\sqrt{r_1}} - \frac{m_2}{\sqrt{r_2}}\right|\left|\frac{1}{\sqrt{r_1}} - \frac{1}{\sqrt{r_2}}\right| |r_1 - r_2| \leq K \ , \tag{3}$$

where $K = M_D/M$ is the dimensionless parameter driving accretion (the only parameter in the model). M_D is the total mass of the disk, while M is the mass of the star.

Previously it has been shown (Balaž et al., 1999abc) that the model leads to the spontaneous appearance of two distinct types of condensate, light and heavy, distinguished by the way they scale with changes of N (the initial number of particles in the disk). The mass of the heavy condensates (planets) were found to be in good agreement with Solar system data and to depend very weakly on the initial mass distribution in the disk. Light condensates were found to obey simple and universal scaling laws. The spin of all condensates obeys a scaling law $s \propto m^\omega$ (in agreement with the planets in the Solar system). It was also shown that certain properties depend strongly on the initial conditions (e.g. initial mass distribution in the disk), most notably the positions of the planets.

Therefore, to determine the spacing of the planets within this effective model it is necessary to know the initial mass distribution in the disk. A general investigation of such initial conditions for accretion has been presented at this conference (Bogojević et al., 2005). As a result we look at the following initial mass distribution in the disk

$$\rho_{in}(r) = \begin{cases} 0 & r < r_* \\ (p-1) M_D r_*^{-1} \left(\frac{r}{r_*}\right)^{-p} & r \geq r_* \end{cases} . \tag{4}$$

Note that the distribution is scale free except for the cut-off r_* (essentially the centrifugal instability). In that paper this distribution was used to analyze the formation of binary star systems. Here we use it to investigate planet formation, in particular the spacing of the planets. In the case of planet formation, depletion of material in the disk crucially affects accretion. The scale r_* is in this case smaller than the depletion length scale (say of the blow off of material in the disk as the result of the star being turned on). The simplest assumption is that all material in the disk is blown off for $r < r_b$ (while nothing is affected for $r > r_b$). For the Sun, r_b is approximately 3 AU, which is much bigger than the scale of the centrifugal instability r_* (around 0.1 AU). The initial mass distribution relevant for accretion is thus

$$\rho(r) = \begin{cases} 0 & r < r_b \\ (p-1) M_D r_*^{-1} \left(\frac{r}{r_*}\right)^{-p} & r \geq r_b \end{cases} . \tag{5}$$

Numerical simulations show that this class of initial conditions leads to a scale free law for $m(r)$. To get $m(r) \propto 1/r^2$ (as for the giant planets in Solar system) we must fix $p = 2.5$. This situation is shown in Fig. 1.

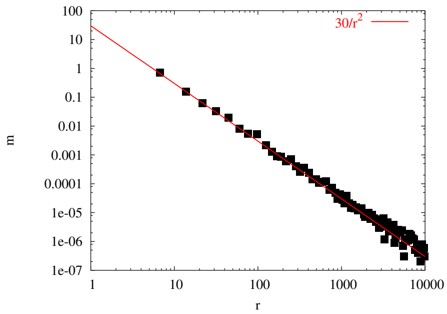

Figure 1: Mass vs. radius for the presented model with $K = 6.6\,10^{-3}$, $p = 2.5$ and $N = 10^7$ initial particles. The condensates fit to a $m \propto r^{-2}$ law.

We have now not only fixed K, the single parameter in our model, but also the parameters r_b and p determining the initial mass density. What remains are direct predictions of the model. In this case we focus on the positions of the planets as a function of their order form the star. Fig. 2 shows that $\sqrt{r_i}$ is a linear function of the index i.

This is precisely the behavior of the outer planets of the Solar system. In addition, the four inner planets also display this kind of spacing. This is shown in Fig. 3. The same is true of the planets of the 55 CnC system. This is an indication how extremely simple effective laws can capture important common features of accretion that hold for different types of planetary systems.

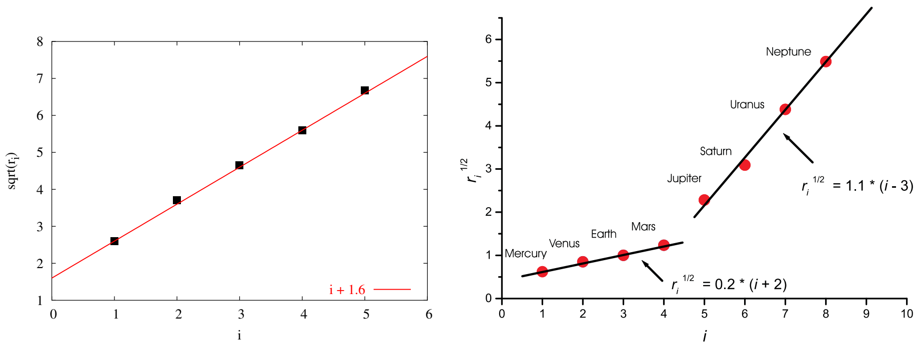

Figure 2: Left: $\sqrt{r_i}$ in the effective model as a function of i (order from the star). Right: Spacings of Sun's inner and outer planets (separately) show the same behavior.

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