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FRACTAL COSMOLOGY TODAY

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Abstract. An overview of the state of arts of the fractal cosmology will be given, with emphasis on the most recent discoveries in the field. New observational evidence will be presented, as well as advances in the theoretical investigations. Particular attention will be paid to the relevant epistemological implications of the concept of hierarchical cosmos, and the status of the model within the present day cosmology will be discussed.

1. PROEM

Contemporary cosmology may be divided into two main sectors. First, it is the observational cosmology (cosmology proper), based on the current empirical evidence via astrophysical data and concomitant theoretical models which match more or less the observational evidence. The other refers to cosmogony (in wider sense) and relies almost exclusively on mathematical constructions, based on some fundamental physical theories, mainly on General Relativity (GR) and quantum field theories (QFT). While cosmological models are being worked out within the so-called Standard Model (SM) and may be considered as a part of positive science, cosmogonical models, even paradigms, are subject to wild speculations, which can be hardly taken for objective science (Grujic, 2006). This qualitative distinction between observational and speculative cosmologies should not be taken as a sign that the first one is immune of difficulties, both methodological and interpretative ones. As we shall se, there is often no direct path between the observed quantities and theoretical construct we call cosmological models. It opens a rather vast space for controversies and disputes among leading astrophysicists and cosmologists of today.

In the next section we shall expose a number of current controversies, paying particular attention to the two principal cosmological paradigms today; (i) homogeneous and (ii) inhomogeneous pictures. The SM belongs to the first class of models, whereas the most prominent representative of the second class is so-called hierarchical cosmos. In section 3 principal features of the hierarchical paradigm are described and the latest literature on the subject reviewed. In section 4 we discuss the observational evidence relevant to deciding between the two paradigms mentioned. In section 5 we address a number of epistemological questions concerning the fractal concept of our cosmos. Finally, in the last section we sum up the principal points of the article.

2. CLAZOMENIAN PARADIGM

Generally, one may conceive two principal classes of universes, finite and infinite ones. The first one appears easy to come to mind, though it is by no means easy to defend. It is this finite universe which has been conceived by all traditional societies and by all so-called ancient civilizations. All but one – Greeks, of course. We shall distinguish two categories of infinite universes: Abderian and Anaxagorian ones (Grujic, 2001). The first one was developed by Leucippus and Democritus in Abdera, the second by Anaxagoras from Clasomenae. Since it happened that it was Clazomenians who founded Abdera in 7th century BC, we shall call both classes of infinite universes Clazomenian paradigm. The first subparadigm postulates that the universe consists of atoms and void and that there are infinitely many worlds like ours, but which may differ considerably from each other. Worlds form and disappear in an eternal interplay of creation and destruction. Abderian universe appears globally homogeneous (and therefore isotropic) for observers like ours. Contrary to this picture Anaxagoras postulated a cosmos constructed on the principle of (discrete) homothetic transformation, which may be stated as follows: everything is a portion of everything. In other words, each part of something resembles the whole, ad infinitum. This is so-called hierarchical cosmos, a primordial variant of modern concept of fractal cosmology.

The fate of both subparadigms resembles somewhat that of the history of the concept of light. They found proponents in the history of the West, alternating or coexisting (Grujic, 2002), just like the concept of particle-like and wave-like nature of light. As we shall see, this sort of indeterminacy has reached us today and the controversy homogeneous versus inhomogeneous is still present in the modern cosmology (Baryshev and Teerikorpi, 2002).

20th century has witnessed the birth of cosmology proper, that is as a positive science. This has been possible due to, first, the rise of the extragalactic astronomy and, second, a remarkable advance of the theoretical studies, based mainly on the General Relativity. The concept of hierarchical cosmos was revived from the very beginning of the last century, and interestingly enough, independently from both advances mentioned. Never-the-less, its development followed the same line as that triggered by GR (Grujic, 2002).

First, it was Fournier dAlbe who resurrected Anaxagorian subparadigm, albeit unconciently, in 1907. This science-fiction cosmology was then put on a sound scientific, astrophysical basis, though in an idealized form, by Charlier (1908, 1922). As in the case of Friedmanns dynamical Einsteinian model, which preceded the observational evidence (primarily due to Hubble, in 1929), Charliers hierarchical cosmos preceded discovery of galactic clustering and superclustering (de Vaucouleurs, 1970). The later two discoveries opened route to enquiries as to the possible structuring based on the self-similarity, which implies that the observable cosmos looks the same at different cosmic scales (scale invariance). And when Benoit Mandelbrot introduced the concept of fractal, the question of the actual cosmic structure was put on a clear mathematical ground. All these discoveries opened, however, Pandora's jar in modern cosmology, which has resulted in what is today known as the Fractal debate. The latter deals with the question: Is the observable universe fractal and if it is up to which cosmic scale this fractality extends? That the answer to the question is far from easy is evident from the fact that the controversy lasts for the last several decades.

3. THE FRACTAL COSMOS

Difficulties facing astrophysicists in discerning actual structure of the (observable) universe are of two principal kinds. First, the very methodology applied in collecting relevant astrophysical data and second, interpretation of the data available. The problem with methods applied is that they appear dependent of the very theoretical models in mind when collecting observables which should decide upon the model assumed. The second difficulty is of a more technical nature, since cosmos is not a laboratory object, where standard experimental technique may be applied.

3. 1. STATISTICAL ANALYSIS

Statistical methods in discerning possible structuring at large scales appear indispensable in cosmological studies (Gabrielli et al., 2005). If the matter distribution turns out inhomogeneous, the question arises as to the possible regularity of the actual distribution. The principal tool for detecting a (quasi)regular distribution is looking for eventual correlations in the cosmic matter spatial distribution (e.g., Sylos Labini et al., 1998). This correlation is revealed by the function

$$G(r) = \langle n(\mathbf{0}) n(\mathbf{r}) \rangle \neq \langle n \rangle^2, \tag{1}$$

where $n(\mathbf{r})$ is density at $\mathbf{r}$, and statistical average is carried out over all occupied points. In the absence of correlations (uniform distribution) the inequality sign in (1) becomes equality. The inhomogeneity region may pass into homogeneous one beyond a distance λ_0 (homogeneity scale), where one can define an average density $\langle n \rangle$. In the cosmic case λ_0 is related to the size of the largest void. Another useful quantity is so-called correlation length (r_c), which separates region where there are fluctuations with respect to the average density and the space without correlations. If the homogeneity scale is finite one has $r_c > \lambda_0$ and the average density is a nonzero positive quantity. Then one can define the correlation function

$$\xi(r) = \frac{\langle n(0) n(r) \rangle - \langle n \rangle^2}{\langle n \rangle^2}, \tag{2}$$

Obviously, this definition works if $\langle n \rangle$ is nonzero, otherwise $\xi(r)$ is infinite. This is exactly the case with an infinite fractal system, for which we have $\langle n \rangle = 0$. Fractal cosmos dissolves so quickly in an infinite space that its average density goes to zero. One is compelled to define another two-point correlation function, which for the fractal system is found to be (Gabrielli et al., 2005):

$$\Gamma(r) = \lim \frac{\langle n(r) n(0) \rangle_R}{\langle n \rangle_R}, \quad R \to \infty, \tag{3}$$

where R is a typical size of the system and the average is taken over all points of the sample taken as origin. The important point to be made is that the whole procedure implies occupied points in the sample. Then, the $\langle n \rangle_R$ is called conditional average density. Another important property of an infinite fractal system is that looking from any occupied point the system looks the same in any direction. This property is called conditional isotropy. The immediate consequence for the fractal cosmos is that will provide the same picture of the sky as the homogeneous universe. If the cosmos around us is fractal, a simple looking into deep space would not reveal it.

Fractal (regular) inhomogeneity appears of a special nature, since at any scale fluctuations of the density (of galaxies, for instance) is of the same order as the (local) average density. In a sense, one has here an essential inhomogeneity, which defies any perturbative method (Sahni and Coles, 1995).

The above statistical considerations concern qualitative differences between (globally) homogeneous and (regularly) inhomogeneous systems. Another important parameter which describes a fractal system is the so-called fractal dimension (Borgano, 1995). In the case of cosmology the simplest way to introduce it is as follows.

One starts from a reference sphere with radius r_0 containing N_0 objects, then within the sphere with $r_1 = kr_0$ one finds $N_1 = k'N_0$, etc. Generally we have the

$$r_n = k_n r_0, \tag{4}$$

for the n-th radius and the number of objects within r would be

$$N(r) = Ar^D, A = \frac{N_0}{r_0^D}, \tag{5}$$

with the fractal dimension of the structure

$$D = \frac{\log k'}{\log k}, \tag{6}$$

(In fact this way of practical estimating the fractal dimension turned out not to be quite appropriate for the general case, in particular for the assumed hierarchical cosmos, but illustrates well the procedure).

3. 2. OBSERVATIONAL TRAPS

In addition to the difficulties in collecting relevant data, as we shall see later on, one encounters the problem of interpreting the existing empirical evidence. Because of the immense vastness of the (observed) universe from one hand and the finite speed of light on the other, we are in position to look at the past of our cosmos. But this opportunity to experience the totality if the space-time cosmic manifold must be paid by necessity to work hard on extracting appropriate information from the data observed. If we adopt the SM, as majority of present-day cosmologists do, then we must be prepared to meet a remote part of the world radically different from that in our immediate vicinity. Further, if one makes averaging a physical quantity, like the density of number of galaxies, this average might differ significantly from a local one, if we cover a region with parts belonging to different cosmic epochs. It implies, also, that relativistic physics (both Special and General Relativity) must be employed in converting the observed quantities from deep space into out terrestrial standards. In particular, one is compelled to solving the null geodesic in order to obtain observational quantities along the past null cone. Many common quantities, like the distance, density etc must be (re)defined when dealing with remote cosmic objects and the final observables may turn out noticeably different for different definitions (Ribeiro, 2001). In the case of uniform-nonuniform density distribution one may observe fractal structure which may be quite compatible with the SM, as argued by Ribeiro (see, e.g., Ribeiro, 2001), in an attempt to reconcile two opposing pictures of the observable universe.

This instance illustrates well the intrinsic entanglement of the theoretical prerequisites and empirical evidence in the case of modern cosmology. Another instance is the possible dependence of the empirical data on the cosmic topology, as argued by a number of researchers in the field (see, e.g., Luminet, 2005, and reference therein). In the case of a multiply connected topology, like that of a torus, for instance, light from distant objects may travel many times around the universe before reaching us. We thus may see a single object many times, just like an object in a hall of mirrors. In other words, we may live in a universe considerably smaller than the observed one, contrary to the common intuition. Hence, before concluding which is the actual shape of our cosmos, we must decide beforehand what is its topology.

Similarly, our statistical methods must be general enough to encompass a possible fractal structure of the observable universe. The standard statistical approach turns out to discard in advance the very possibility of such a structuring and can not be used in discerning hierarchical from uniform distribution (Gabrielli et al., 2005). The use of an inappropriate method has been the principal source of the current controversy uniform versus fractal cosmos, which is dubbed The Fractal Debate (see, e.g., http://pil.phys.uniroma1.it/debate.html).

4. PRESENT-DAY SITUATION

We give a brief overview of the actual state of affairs concerning the search for the real cosmic structure. We first discuss the recent theoretical advances and then give a short review of the observational evidence.

4. 1. THEORETICAL INVESTIGATIONS

Theoretical studies relevant to estimating the actual matter distribution within the observable universe can be divided into several research areas.

(a) *Mathematical studies.*

It is well known that fractal structures are not amenable to the standard mathematical analysis, first of all to the calculus. For instance, fractal (plane) curves appear not to be differentiable. However, the standard analysis has been extended to a more general form, which allows structures like fractal ones to be dealt with. This research line was initiated by the very (co)founder of calculus, Leibniz, who introduced the concept of the fractal derivative and integral. The concept was further developed by the most eminent mathematicians, like Laplace, Riemann, Liouville, Heaviside, Erdelyi, etc. The most general forms of (fractal) integral and derivative are:

$$ {}_aD_x^{-\alpha} = \frac{1}{\Gamma(\alpha)} \int_a^x (x-y)^{\alpha-1} f(y)dy, \;\; a \leq x, \tag{7} $$

$$ {}_aD_x^{\alpha} = (d^n/dx^n)\,{}_aD_x^{\alpha-n}, \tag{8} $$

They reduce for integer α to the ordinary definition of the function integral and derivative, respectively. Recently, a monograph on the subject has been published (West et al., 2003).

(b) *Statistical investigations*

Initiated by the paper by Pietronero (1987) statistical analysis of the cosmological distributions has been tackled in a qualitatively different way as compared with those methods applied in the standard statistical investigations. These methods are applicable for any mean density of the system, including zero-density, and allow one to deal with an infinite fractal system, as elaborated in the recently published monograph (Gabrielli et al., 2005). In particular one demonstrates rigorously that the famous Seeliger-Neuman's paradox is resolved for an infinite fractal system with the (fractal) dimension $D \leq 2$.

Further generalizations of the simple fractal model have been investigated too. First, a more general case of mixed systems with components (subsystems) forming fractal structures with different fractal dimensions is further elaborated. Studies of higher order correlations functions, like three-point correlations, have been started, as another way of refining the statistical analysis. Also, other system parameters, which determine the fractal system beyond the very fractal dimension, like the so-called lacunarity, are investigated (e.g., Sylos Labini et al., 1998).

(c) *Physico-theoretical modeling*

A proposed physical model of a presumed structure must go beyond very descriptive level, if it pretends to be realistic and intend to be accepted by the relevant part of the scientific community. The problem belongs to the issue of dynamics of complex systems (e.g., Mainzer, 1997; Solomon and Shir, 2003), more precisely to dynamics of hierarchical structures (e.g., Nicolis, 1986). In the case of cosmic structure this means that a model which explains how the presumed fractal structure is formed must be offered. Do we have a plausible model of a self-gravitating self-organizing cosmic matter? Many efforts have been made in this direction and a number of plausible results have appeared in the recent literature, in particular those based on the notion of self-organized criticality (e.g., Sylos Labini and Pietronero, 2001). We mention first the work by Combes (1999), which deals with general case of the self-gravitating (sub) units. The remarkable result of her investigations is that the model employed explains equally well clustering of the galaxies in the deep cosmic space and the same for the interstellar galactic dust. This result illustrates the best the essential feature of the self-similarity - nonexistence of a referent level. The (quasi)atomic unit may assume various physical contents, be it a grain of dust or a galaxy, or a cluster of galaxies, the mechanism driven by the ruling force (gravitation in this case) should be approximately the same.

4. 2. OBSERVATIONAL DATA

As mentioned above, one must be aware of the tight link between the data observed and the method of collecting them, if a realistic picture of the cosmic structure is to be achieved. Further, the very interpretation of the observables should eliminate from the start a possible bias toward a particular cosmological model. In our case it is, as a rule, an implicit assumption that the universe is uniform at sufficiently large cosmic scales.

The principal parameter to be estimated from the observations is the fractal dimension D. Practically it reduces to estimating how the mean number density of galaxies varies with the volume of a sphere with the centre at the observer (on Earth). It can be shown that this quantity obeys the law

$$\langle n \rangle \propto r^{D-3}, \;\; 0 < D \leq 3, \tag{9}$$

In the case of a uniform (Poisson) distribution (homogeneous universe) $D = 3$ and the average density is constant. Otherwise it goes down and in the limit of an infinite sphere ($r \to \infty$) becomes zero. The latter situation is just the case of a fractal universe. Hence, the crucial question appears to be: what is the actual fractal dimension of the observable cosmos?

Numerous estimates from the large number of galactic catalogues point to the estimate $D = 2 \pm 0.2$, within distance of approximately $50h^{-1}$ Mpc. This $D \approx 2$ might have remarkable cosmic significance. This is the border value which just ensures, within the static hierarchical cosmos, the compact projection of the galaxy spatial distribution onto the celestial sphere, what is tantamount to the isotropy as observed on the sky. At the same time, it just ensures the cosmic stability with respect to the self-gravitational pull, as mentioned before in the context of the Seeliger-Neumann paradox.

This result, however remarkable, should be considered within the principal issue of the cosmic structure. We know there exist galactic clustering and superclustering, but what lies beyond it? Is the universe hierarchically designed at all scales and if it is what would be consequences of it? Hence, the main preoccupation of the astronomers interested in the large-scale structures is determination of the homogeneity scale λ_0. Currently this is used as a free parameter in fitting various observables to theoretical results.

One of major problem faced by the (standard) statistical analysis of the red-shift catalogues is the so-called galaxy-cluster mismatch. Namely, it turns out that the fractal dimension for the galaxy distribution is different from that for the clusters of galaxies. This mismatch has been successfully resolved within the concept of the (integrated) conditional density, as shown in Fig. 1, for the Abell clusters (e.g., Gabrielli et al., 2005). The mismatch is easily explained as the finite-size effect in the relevant catalogues of galaxies and clusters.

That the fractal dimension $D = 2.1$ agrees with that of galaxy distribution corroborates the validity of the approach and self-consistency of the physical model used.

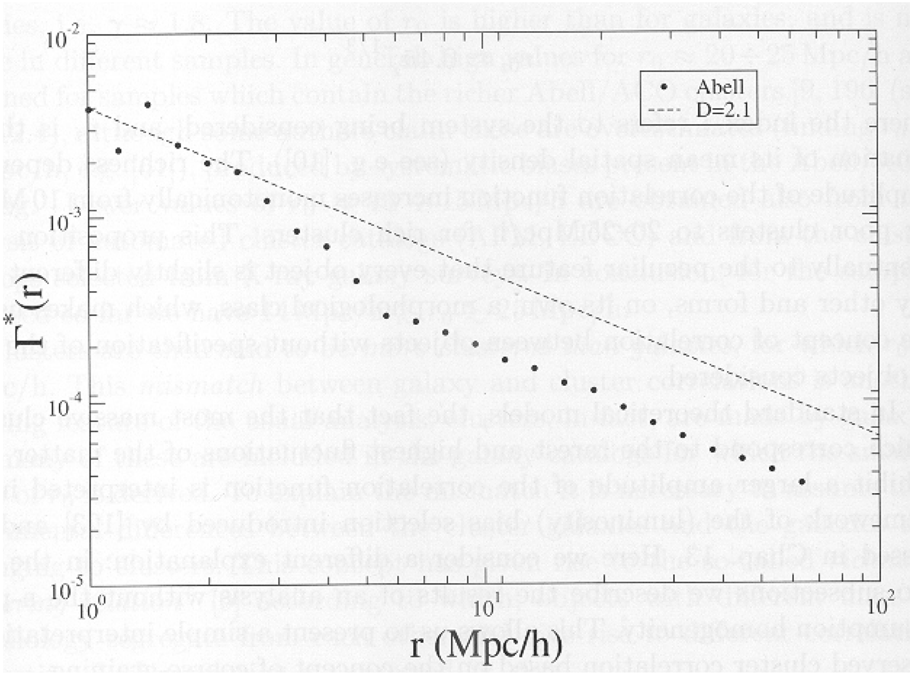

Figure 1: The integrated conditional average density for the Abell clusters.

4. 3. DYNAMICAL MODELS

So far we have been dealing with statistical pictures of the static cosmic structures. The first Charlier model was purely static, though he was aware of the first red-shift observations. As we know the latter led to the concept and model of an expanding universe, which forms an essential part of the Standard Model. The question arises whether the concept of hierarchical cosmos is compatible with cosmic expansion. The answer is positive. In a paper by Joyce *et al* (2000) it is shown that even if the scale of the crossover to homogeneity goes to infinity, the fractal structure appears compatible with the overall expansion of an open universe. Fractal component can be treated as a perturbation to the CBR homogeneous background and the issues of nucleosynthesis and structure formation can be addressed.

Not only the fractal subparadigm can deal with the cosmic expansion, but even with the recently observed accelerating one. In a recent paper (Grujic, 2004) it is argued that the Charlier's model, combined with the dark energy hypothesis, can explain, at least qualitatively, the attractive interaction between nearby celestial objects and repulsive one between remote cosmic subsystems, like galaxy clusters.

5. SOME EPISTEMOLOGICAL QUESTIONS

We start with the basic question regarding the fractal paradigm: How fundamental the issue of the actual structuring of the observable universe is? To illustrate the point, let us quote some issues, arising in contemporary cosmology, in particular within the SM. (i) Is SM the only respectable among all offered on the market? (ii) If it is, is our universe open or closed? (iii) Is the very structure of the (observable) cosmos relevant to the previous question?

(i) Although it is widely accepted, SM is by no means the only that has been investigated, but we shall not dwell on it here (see, e.g. Grujic, 2006).

(ii) The issue appears still open, although the current consensus revolve around the flat (Euclidian) cosmic space.

(iii) Though no specific calculations have been published on the matter, this question is going to be raised. The ruling eschatological picture is based on the uniform matter distribution. If we accept the fractal structuring, at least within the observable cosmos, may result in a future universe different from that of homogeneous one. There may be essential difference, for that matter, between Abderian and Anaxagorian (sub)paradigms.

We mention here that the SM imposes stringent limits to the extent of fractal structuring, because of the finite age of the cosmos. What does not prevent further higher order clustering in the future.

Finally, we mention the so called Hubble-de Vaucouleur's paradox, which puzzles modern cosmologists. Namely, de Vaucouleur's (hierarchical) picture of the cosmos covers the scales from 1 to 200 Mpc. At the same time we know that Hubble law holds from distances starting with 1 Mpc. Since the latter has been considered to be an essential outcome of the homogeneity cosmological principle (assumption) the question arises as to how to reconcile these two opposing pictures. The paradox bears number of epistemological implications, which remain to tackle in the future. We just mention that the cosmological gravitational red shift for fractal structure with dimension $D = 2$ yields the linear redshift-distance relation. We shall not dwell on it here, but direct interested readers to the paper by Baryshev (2000).

6. SUMMARY

We have shown that the fractal cosmology has provided a realistic alternative to the homogeneous cosmos assumption, with fractal dimension $D \approx 2$, within scales from 1 to 200 Mpc. The latest theoretical investigations support this picture and also provide a reasonable explanation why the homogeneity assumption has been wrongly considered as the only valuable cosmological principle. The open questions that remain to be answered are the extent of the fractal structure, as well as the more detailed mechanism which this structure generates. Finally, a number of epistemological issues remain open to further considerations, in particular those pertaining to the concept of an infinite universe, the idea we owe to Pre-Socratics, which we have dubbed Clazomenian paradigm.

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